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Optimal Positioning of Unmanned Aerial Vehicle (UAV) Base Stations Using Mixed-Integer Linear Programming

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Abstract: In wireless communications, traditional base stations act as the backbone for providing network connectivity to users. These base stations, however, require significant resources to construct and are therefore not suitable for remote areas and disaster scenarios. This challenge makes them unfit for deployment in remote areas or in disaster scenarios where fast network establishment is necessary. To address these challenges, cellular base stations installed on Unmanned Aerial Vehicles (UAVs) can be an alternative solution. UAVs provide quick deployment capability and can adapt to changing environmental situations, making them ideal for dynamic network scenarios. In this paper, we address the critical issue of UAV positioning to maximize the total user coverage, which can be formulated as a mixed-integer linear program. Given the complexity of larger-scale scenarios related to the number of users, we suggest a two-step method. First, we group users into clusters, and then we optimize the UAV positions with respect to these clusters. This approach introduces a trade-off between computational time efficiency and optimality, which can be tuned by adjusting the number of clusters. By varying the number of clusters, we balance computation time with the optimality of the UAV locations, allowing flexible deployment in diverse scenarios.

Keywords: UAV; MILP; communication; optimization; clustering

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1. Introduction

Unmanned Aerial Vehicles (UAVs) are used to perform a variety of tasks in commercial, military, and academic domains with applications including disaster response and weather monitoring, among others [1]. In contrast to the dynamic nature of UAVs, traditional base stations in wireless communications are stationary. Moreover, static base stations are costly and time-consuming to construct [2]. Motivated by applications requiring wireless networks in remote areas, emergency scenarios, and crowded areas (such as large sporting events), we consider UAVs equipped with mobile base stations [3]. Such UAV base stations (UAV-BSs) provide several advantages in providing cellular and network connectivity to the users [4]. The positioning of the UAV-base stations is important to provide effective communication and strong signal strength [5].

Air-to-ground (A2G) networks enable air-to-ground communications. An example of this is communication between an aircraft and a ground station. In an A2G network with multiple UAV Base Stations (UAV-BS), the optimal control of the UAV-BS position and communication range is a key design factor that determines the overall network performance, measured based on signal strength, mobility, performance, coverage and several other measurement factors [5]. There are several advantages to deploying UAVs in disaster areas where traditional base stations may be inefficient. Stationary base stations are unreliable in such situations. In such scenarios, these UAVs could provide a solution to the problem by acting as temporary mobile towers and providing the necessary network connectivity to ground users [6]. UAVs can provide assistance by providing connectivity for data transfer between ground users and supporting emergency services with sensors to monitor the scenario [7].

In this paper, we consider the problem of placing a group of UAVs within their environment in order to maximize user coverage. The relevance of this work is shown by the increased demand for efficient communication methods during critical scenarios where the deployment of traditional base stations is inefficient. The aim of this research is not only to position the UAVs optimally but also to provide a scalable framework to adapt to various user densities. UAV base stations are employed as an alternative in remote areas, emergencies, and areas with high population density. For given user positions, we show how to formulate this as a mixed-integer linear program (MILP) [8]. The MILP formulation is chosen because it effectively handles both continuous variables (UAV positions) and binary decisions (user associations) while providing optimal solutions for UAV placement to maximize coverage. As finding the optimal solution does not scale well with the number of users, we also propose a heuristic algorithm based on clustering, where the number of clusters is a tunable parameter that trades off computational complexity with optimality.

Main Contributions

The main contributions of this paper are as follows.

- The formulation of UAV positioning as a mixed-integer linear program (MILP) to maximize user coverage in wireless communications.
- The development of a clustering-based approach to address scalability issues for scenarios with large numbers of users.
- The introduction of a tunable parameter (number of clusters) to trade off computational complexity with optimality.
- The demonstration of how the number of clusters can be used to balance user coverage and computation time through experimental results.
- An analysis of the computational time and user coverage as functions of the number of UAVs and clusters.

We describe our system model in Section 3 and then formulate the problem of finding the optimal UAV positions in Section 4. We extend our algorithm using clustering in Section 5, present simulation results in Section 6, and conclude in Section 7.

2. Literature Review

Combining multi-UAV platforms requires complex strategies to apply in real-world environments [9]. The main challenge in this type of communication is to develop a strategy to deploy UAVs to meet the needs of users wirelessly and meet the demands of network traffic [10]; see the survey [11]. A machine learning approach is used to simulate scenarios with random user positions operating in large-scale environments. The transmission power is minimized to improve the environment [12]. For uplink and downlink coverage, a probabilistic method is used where the user distribution follows randomness in the long run. Using UAVs for damage control as well as defense requires an efficient central controller to keep all interconnected UAVs under control [13]. A network and security architecture is proposed in which public and private keys are assigned to a centralized UAV communication network. In this architecture, a central ground station collects data from deployed UAVs, and communication is carried out over short distances. The energy limitations of UAVs, flight time, and computational power are issues faced by this type of network. Ref. [14] describes providing wireless coverage to ground users in disaster scenarios. It describes the UAV's ability to analyze the ground user's location and demonstrates its efficiency in network coverage. The UAV is integrated into a cellular system to communicate with other devices and provide a reliable network connection. Search and rescue missions require tolerance to latency and jitter, as well as maximum throughput [7]. The UAV communication infrastructure must take these parameters into account. In [7], UAVs are deployed in a UAV-based WiFi network to support search missions. The flight zone of the UAV is discretized, and the UAV is controlled to fly based only on 3D grid coordinates. We also aim to cover the maximum number of users by limiting the number of drones using a comprehensive search algorithm.

The UAV base station provides network coverage in areas where traditional base stations have insufficient network coverage. Although this type of network connection brings high benefits, it also poses the problem of optimizing the location of the base station [6]. UAV communication faces several challenges in terms of effective ground communication with users and maximizing user coverage [2]. Requiring UAVs to cover larger areas is a key concern. UAV mobility can cause communication issues due to the dynamic network topology that depends on how UAVs are interconnected. Too much maneuverability of UAVs is an important factor to consider, as it can prevent them from being localized according to the user's location, leading to limitations in UAV performance. Ref. [15] proposes a polynomial-time algorithm and K-means clustering to minimize mobile base stations and coverage for a group of ground users by ensuring that each ground user is within the communication range of one of the mobile base stations. These base stations are arranged in a spiral toward the center according to the range of uncovered ground users. This solves the problem of avoiding excessive movement of UAVs. Some applications that need to ensure the quality of service to sensitive users require the optimization of other parameters such as the total bandwidth and total transmission power of the UAVs [16]. The shared power allocation is used to communicate securely with users under a specific system configuration [17]. To protect the communication between the UAV-BS and legitimate ground users in critical security areas, a robust common UAV trajectory is developed, considering simulations with multiple eavesdroppers and specifying the confidentiality of data transmissions in [18]. The problem of maximizing throughput is also one of the challenges in mobile relay systems, which requires the optimization of relay trajectories and the optimization of source or relay power allocation [19]. These are some of the challenges in establishing UAV-based communications in multiple domains, such as disaster management and secure military communications. Our work focuses on one of these challenges, the optimization of UAV positioning, which can indirectly solve other problems as well. A goal-oriented communication framework is proposed in [20] using deep reinforcement learning with a proactive repetition scheme to optimize the control, data selection, and repetition for the real-time tracking of targets by UAVs. Ref. [21] proposes a multi-agent reinforcement learning approach to optimize UAV communication systems to optimize energy efficiency and integrates distributed ledger technology to enhance scalability and security. Ref. [22] aims to minimize the mission completion time by optimizing the UAV-IRS trajectory and transmission power using deep reinforcement learning.

A centralized UAV positioning strategy is employed in [23]. Here, the UAVs are controlled by a central controller and are used as flying access points to form a mesh network. This network provides connectivity to ground nodes. The paper is focused on minimizing the total number of UAVs while maximizing the data rate requirements. Software-defined networks are increasingly used in managing wireless communication networks [24]. This paper proposes a centralized learning approach to deploy UAVs in disaster areas and achieve maximum throughput. Simulations using the algorithm in the centralized approach prove that it is suitable for finding the optimal position of UAVs in disaster environments. Ref. [25] compares centralized and decentralized multi-robot coverage where ground robots are controlled under the supervision of a UAV. Here, the UAV acts as a central controller to control the movements of all the robots. Ref. [26] proposes a centralized approach to address the problem of resource allocation and UAV trajectory design. The main goal is to maximize the overall user coverage of ground users. The authors also suggested that the size of the overall system model is proportional to the number of constraints on the problem, which makes the computations more complex and makes it difficult to transfer information, decisions, or actions between UAVs. This correlates with the current need to compare and contrast other optimization algorithms to overcome these issues. Table 1 summarizes key findings from the selected existing literature, identifies gaps in the research, and outlines proposed solutions to address those gaps.

Table 1. Summary of recent works on UAV communication and optimization.

Study	Key Findings	Gaps Identified	Proposed Solutions (Related to Our Work)
Wu et al. (2024) [20]	Develop a goal-oriented communication framework to optimize the control of a UAV in a target-tracking scenario using deep reinforcement learning.	The framework is specific to a single UAV and the target tracking scenario.	We consider a group of UAVs, each of which acts as a base station in an ad hoc communication network, with the goal of positioning the UAVs to maximize network coverage to ground users.
Saleh et al. (2024) [22]	Optimize trajectories of UAVs in 6G Terahertz networks using deep reinforcement learning to minimize time to task completion.	Optimality of the generated trajectories is not guaranteed, and the objective is solely based on completion time.	We propose finding the provably optimal UAV locations. For trajectory planning problems, our approach could be used to find optimal positions at each point in time.
Liu et al. (2022) [4]	Use deep reinforcement learning to find UAV policies for maximizing data transfer from ground base stations to a data center.	As this is a *nonlinear* optimization mixed integer optimization problem, the policies found may be suboptimal.	We obtain optimal (static) UAV positions for the problem of maximizing user connectivity, as opposed to transferring data to a data center.
Ali et al. (2024) [21]	Use multi-agent deep reinforcement learning to position UAVs in an ad hoc communication network.	As the UAV positions are found using reinforcement learning, they may be suboptimal.	We find the provably optimal UAV positions by solving a mixed integer linear program.

3. System Model

The system model shown in Figure 1 represents a network of UAVs and their environment. In this section, we explain the overall model framework, including the dynamics between UAVs and ground users. Furthermore, the environment consists of mathematical principles that play a role in the assignment of users to UAVs and the user distribution framework.

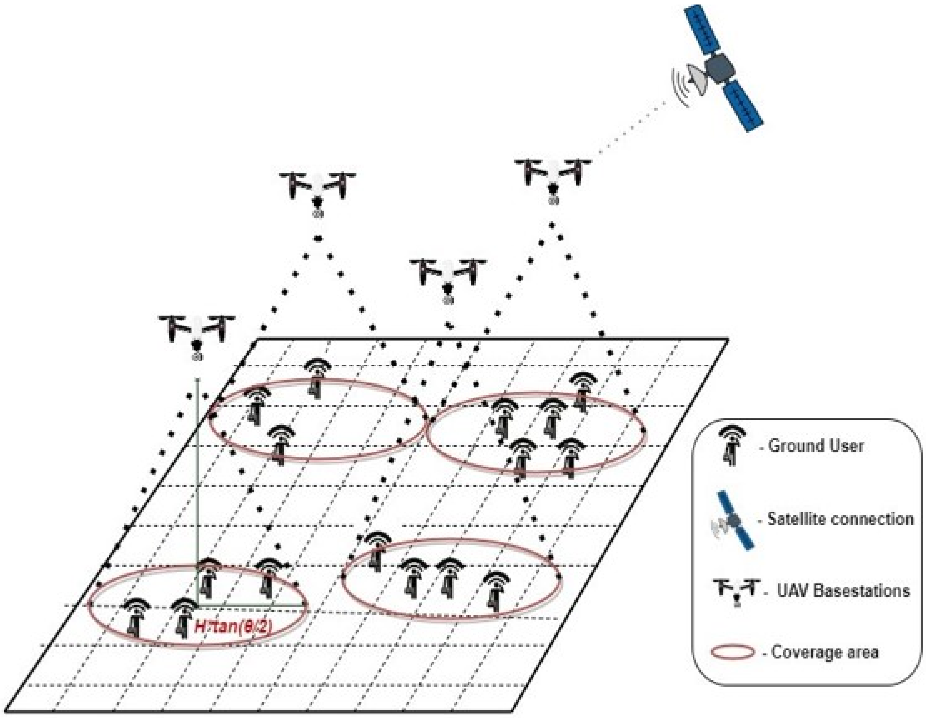

Figure 1. Overall system model, where H is the UAV altitude and $H \times \tan(\theta/2)$ is the coverage radius.

The UAV environment is designed to reflect the limitations and simulate the general functionality of the system. The total area is divided into a grid, $X_{\text{cov}} \times Y_{\text{cov}}$. A set of N_{users} is distributed on this grid. The UAVs operate at a constant distance above the ground, and we assume that all UAVs in the area fly at a constant altitude H_{UAV}. The Cartesian coordinates of UAVs and users refer to this grid. They are considered to be at any grid point. We assume that the UAV can communicate via satellite and act as a communication bridge between the user and the satellite. This type of connection is necessary in scenarios such as disaster relief and military operations, where it may be difficult to establish a ground terminal.

3.1. Resource Allocation

Let M denote the number of UAVs and N denote the number of users. Each UAV has a certain bandwidth. In this scenario, BW_{UAV} is assumed to be 4 MHz. This 4 MHz bandwidth is selected based on a realistic bandwidth value that can reproduce the actual limitations depending on the operating conditions. The bandwidth of a UAV with the reduction factor is expressed as,

$$BW_{\text{effective}} = BW_{\text{UAV}} \times \text{Reduction Factor}. \tag{1}$$

This reduction factor is considered based on practical constraints such as channel conditions, hardware limitations such as transmitter and receiver efficiency, etc. The total bandwidth of the UAV is divided into resource blocks, and each resource block is assigned to a ground user. The bandwidth of each resource block is 180 kHz. Considering the total bandwidth of each UAV and the reduction factor, the effective bandwidth is

$$BW_{\text{effective}} = 4\,\text{MHz} \times 0.9 = 3.6\,\text{MHz}. \tag{2}$$

The reduction factor of 0.9 means that the system can maintain 90 percentages of the theoretical bandwidth after considering practical limitations. Taking the reduction factor into account, the total bandwidth of each UAV is reduced to 3.6 MHz. Since the bandwidth of each resource block is $B_{\mathrm{RB}} = 180$ kHz, the total number of resource blocks N_{RB} that each UAV can accommodate is

$$N_{\mathrm{RB}} = \frac{BW_{\mathrm{effective}}}{BW_{\mathrm{RB}}} = \frac{3.6\,\mathrm{MHz}}{180\,\mathrm{kHz}} = 20. \tag{3}$$

If each resource block is assigned to a user, the total number of users that each UAV can accommodate is 20. The coverage of UAVs with respect to the ground users is shown in Figure 2.

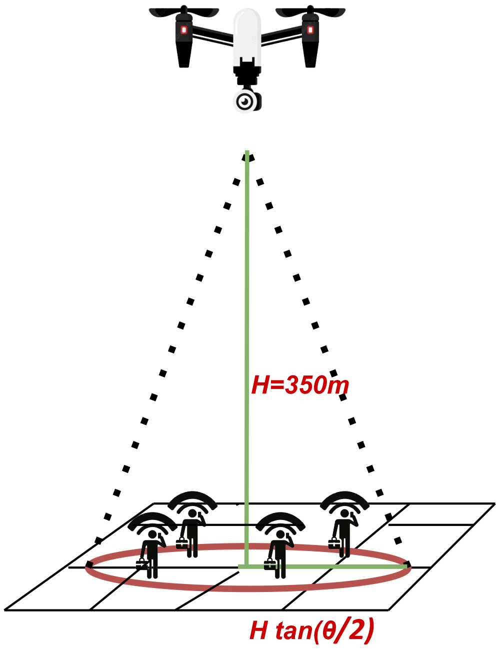

Figure 2. UAV coverage over ground users.

3.2. Ground User Distribution

We assume that the ground users are grouped into certain areas called hotspots. Hotspots are locations with a dense number of users and depict urban areas. The hotspot center is the center point of the radius where users are located. The users are randomly distributed such that each hotspot has a certain number of users, and a small percentage of the total users are evenly distributed across the environment depicting the rural areas. This distribution model mimics the real-world scenario where there is a dense number of ground users in urban areas such as cities and fewer users in rural areas. These hotspots allow efficient resource allocation and the targeting of service improvement. This user distribution also ensures that both the urban and the rural network needs are considered in the system design. Figure 3 illustrates the distribution of ground users according to hotspots, where the users are distributed according to Algorithm 1.

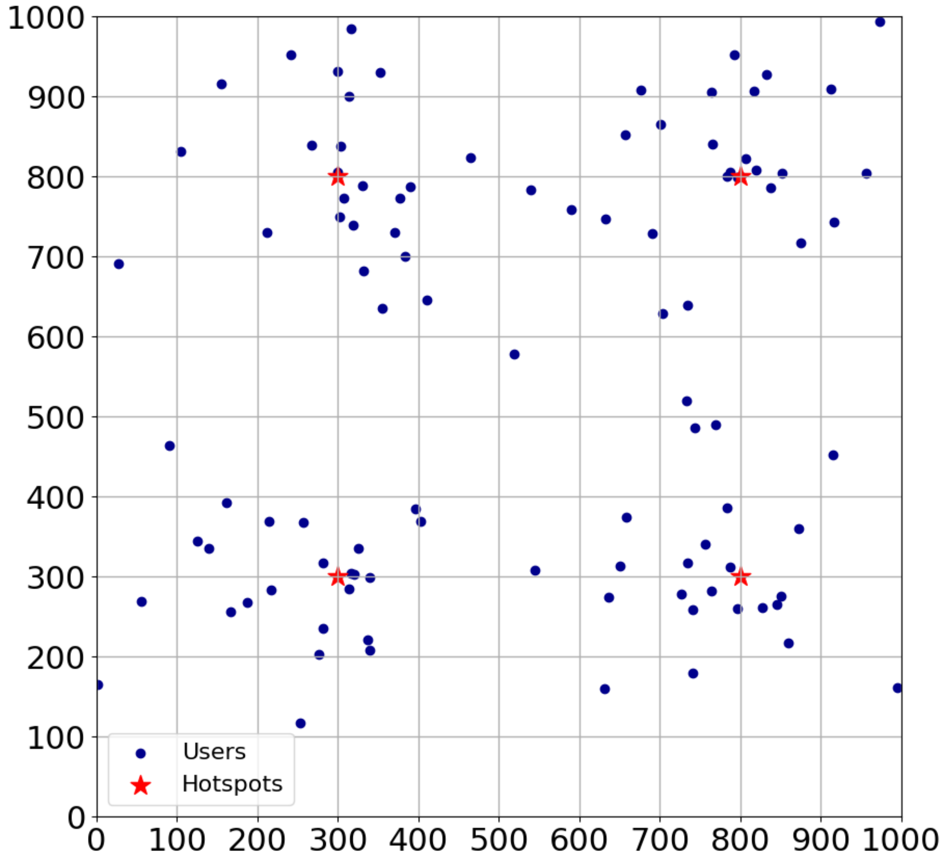

Figure 3. Ground user distribution. Blue circles indicate users, red stars indicate hotspots, and the axes are the coordinates in meters. The users are randomly distributed near the hotspots according to Algorithm 1.

Algorithm 1 Algorithm for user distribution based on hotspots

Initialization:
Inputs: Number of users n, number of UAVs m, hotspot radius r_{hotspot}, hotspots T
$D \leftarrow \lfloor \frac{n}{m} \rfloor$ ▷ D - users per hotspot
Initialize L as an empty array to store user locations
for each hotspot t in T **do**
  for d from 1 to D **do**
    Generate random r uniformly within $[-r_{\text{hotspot}}, r_{\text{hotspot}}]$
    Generate random ϕ within $[0, 2\pi]$
    $x \leftarrow r \times \cos(\phi) + t_x$ ▷ t_x, t_y - coordinates of hotspot center
    $y \leftarrow r \times \sin(\phi) + t_y$
    Append (x, y) to L
  end for
end for
Generate random locations for remaining $(n - D) \times |T|$ users and append to L
return L

3.3. User Association

Ground user association is the process of determining whether a user is connected to a UAV. The connection between a user and a UAV depends on various factors. The assignment of a user to a UAV depends on the distance between the user and the UAV. The distance between the user j and the UAV i is calculated using the Euclidean distance formula, which is formulated in two-dimensional space: The calculation of the distance between a user and a UAV is as follows:

$$d_{ij} = \sqrt{(x_{\mathrm{UAV},i} - x_{\mathrm{user},j})^2 + (y_{\mathrm{UAV},i} - y_{\mathrm{user},j})^2} \tag{4}$$

where $x_{\mathrm{UAV},i}$ and $y_{\mathrm{UAV},i}$ represent the Cartesian coordinates of UAV i in the grid, and $x_{\mathrm{user},j}$ and $y_{\mathrm{user},j}$ represent the coordinates of user j. The distance formula is used together with the coverage radius of the UAV. The coverage radius determines the limit of UAVs that can cover a user. When a user is within the detection range of a UAV, the distance between them is calculated. If the distance between the user and the UAV is less than the detection range, the user sends a connection request to the respective UAV. User j sends a connection request to UAV i when

$$d_{ij} \leq R_{\mathrm{cov}} \tag{5}$$

where d_{ij} is the distance between user j and UAV i and R_{cov} is the coverage radius of the UAV. The maximum capacity of each UAV is 20 users based on the available resource blocks and maximum user capacity. Resource blocks are assigned to users based on distance, and priority is given to users closest to the UAV. Table 2 describes the simulation parameters used in this paper.

Table 2. Symbols and simulation parameters

Description	Symbol	Value
Number of UAVs	m	
Number of users	n	
Hotspots	T	
Hotspot radius	r_{hotspot}	200 m
Users per hotspot	D	
Position of UAV i	$(x_{\mathrm{UAV},i}, y_{\mathrm{UAV},i})$	
Position of user j	$(x_{\mathrm{user},j}, y_{\mathrm{user},j})$	
Center of cluster j	$(x_{\mathrm{cluster},j}, y_{\mathrm{cluster},j})$	
Distance between UAV i and user j	d_{ij}	
Coverage angle	θ	$\pi/3$ rad
Coverage radius	R_{cov}	202.07 m
Altitude of UAV	H	350 m
Bandwidth of each UAV	BW_{UAV}	4 MHz
Effective bandwidth	$BW_{\mathrm{effective}}$	3.6 MHz
Resource block bandwidth	BW_{RB}	180 KHz
Resource blocks per UAV	N_{RB}	20
Grid size	-	100 m
UAV–user allocation variable	X_{ij}	
UAV–cluster allocation variable	Y_{ij}	
Number of users in cluster j	U_j	
Big-M method parameter	M	2000 m
Binary auxiliary variable	ϵ_{ij}	

4. Optimization Problem Formulation

We now show how to formulate the problem of placing the UAVs to maximize the number of users served as a Mixed-Integer Linear Program (MILP).

4.1. Decision Variables

The decision variables in the optimization problem are the UAV positions $(x_{\mathrm{UAV},i}, y_{\mathrm{UAV},i})$ along with a set of binary variables X_{ij} that are one if UAV i is allocated to serve user j and zero otherwise. The presence of both real and binary variables makes this a *mixed-integer* optimization problem. The decision variables are important in determining the optimal placement of UAVs while ensuring efficient resource allocation and maximizing user coverage. The binary variables in user association enable clear user assignment and also prevent overlaps in service allocation.

4.2. Objective Function

The objective of the optimization problem is to maximize the total number of ground users that are served by the UAVs. Intuitively, we expect this to place UAVs in regions with dense user populations. This objective of maximizing the total number of users ensures optimal coverage while considering the service capacity of the UAVs and the other constraints in UAV deployment. This also provides flexibility in adapting to the changes in user distribution depending on the user density in the real world. This objective function allows efficient resource allocation and balances the trade-off between the coverage area and the service quality. In terms of the user associations, the objective is to maximize

$$\sum_{i=1}^{n} \sum_{j=1}^{m} X_{ij}. \tag{6}$$

Note that this objective function treats all users equally; we could instead modify the objective to bias the UAVs towards certain groups of users (e.g., users who pay more for the service) to enhance connectivity for that group.

4.3. Constraints

We now describe the constraints on the decision variables. The first constraint ensures that each user j is associated with at most one UAV, which is represented as

$$\sum_{i=1}^{n} X_{ij} \leq 1, \quad \forall j \in [\![1, m]\!]. \tag{7}$$

The notation $[\![1, m]\!]$ denotes the set $\{1, 2, \ldots, m\}$ and m is the number of users. This constraint ensures that each user has at most one network to access.

The next constraint is the maximum user capacity constraint, which is to make sure that for each UAV i, the maximum number of users that each UAV can serve is 20. This value is the representation of the bandwidth limitation of each individual UAV. The maximum user capacity constraint is

$$\sum_{j=1}^{m} X_{ij} \leq 20, \quad \forall i \in [\![1, n]\!]. \tag{8}$$

The boundary constraint is the one that makes sure that no UAV exceeds the boundary limit of the environment. This boundary is the area where the network service is provided. The boundary condition is represented as

$$R_{\text{cov}} \leq x_{\text{UAV},i} \leq L - R_{\text{cov}} \tag{9}$$

$$R_{\text{cov}} \leq y_{\text{UAV},i} \leq L - R_{\text{cov}}, \quad \forall i \in [\![1, n]\!]. \tag{10}$$

where $L \times L$ represents the length of the environment.

Next, we impose the constraint that a UAV is able to serve a user only when the user is within the coverage radius of the UAV. As this is an implication, it is not directly implementable as a linear constraint. Using the Big-M method, however, we can formulate the constraint as

$$\sqrt{(x_{\text{UAV},i} - x_{\text{user},j})^2 + (y_{\text{UAV},i} - y_{\text{user},j})^2} \leq R_{\text{cov}} + M(1 - X_{ij}) \tag{11}$$

where the parameter $M > 0$ is a large number. When $X_{ij} = 1$, this constrains the position of the UAV such that the user is within its coverage radius. When $X_{ij} = 0$, the parameter M is large enough that this poses no constraint on the UAV position.

5. Clustering

While the MILP formulation may be used to find the optimal placement of the UAVs, the computational complexity does not scale well with the number of users (as illustrated in Section 6). For scenarios with large numbers of users, we therefore propose first clustering users into groups and then solving an optimization problem to place the UAVs with respect to the clusters. Let k denote the number of clusters, U_j the number of users in cluster j, and $(x_{c,j}, y_{c,j})$ the center of cluster j. An illustration of the clustering of users is shown in Figure 4.

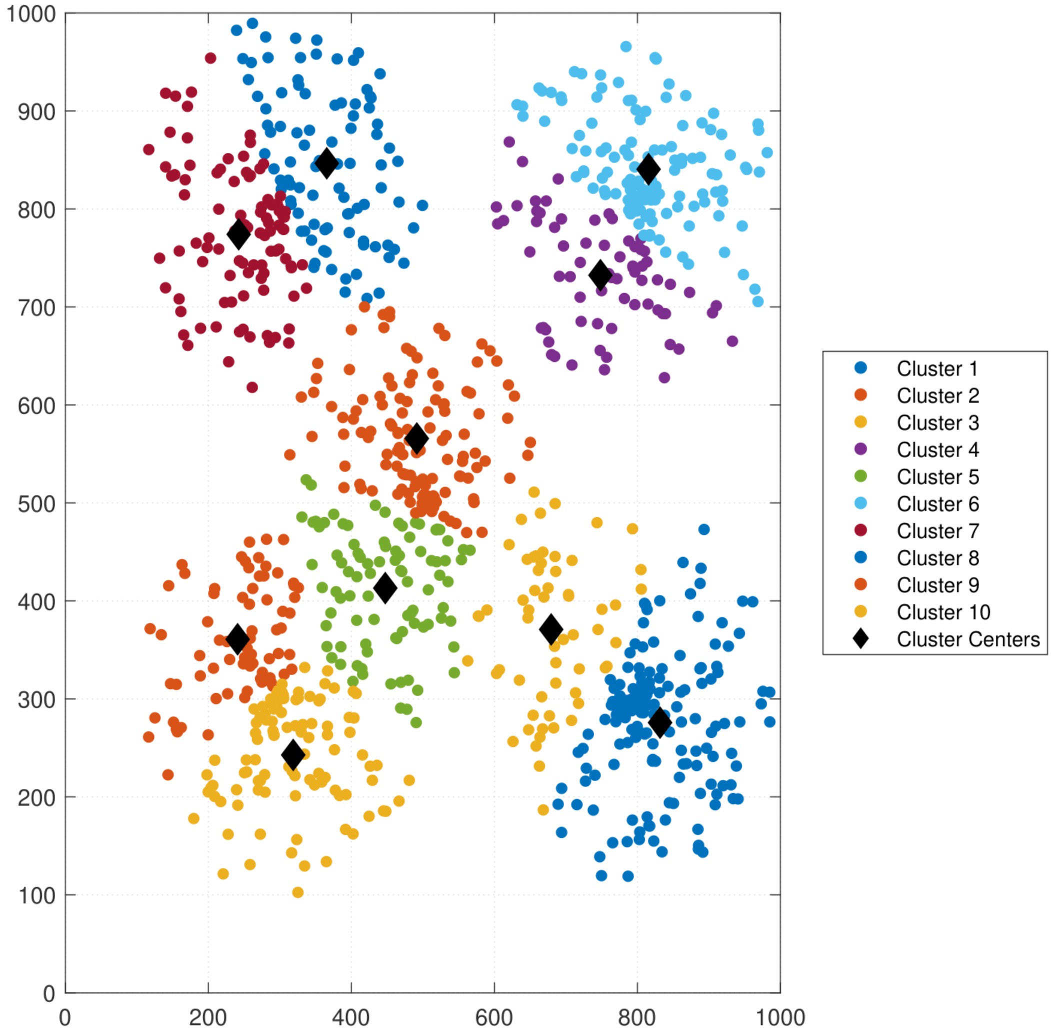

Figure 4. Clustering 1000 users into 10 clusters using k-means clustering. The axes indicate position in meters.

5.1. Decision Variables and Objective Functions

In the previous optimization problem, the user association variables were binary as each individual user was either served by a particular UAV or not. In the clustering approach, the user association variable Y_{ij} is the *percentage* of users in cluster j that are served by UAV i. This formulation allows a UAV to serve only part of a cluster (e.g., when the cluster has more users than the capacity of the UAV). Here, Y_{ij} is continuous and provides more flexibility in resource allocation compared to the binary variables by implementing partial service allocation within the clusters. This allows efficient usage of UAV resources to serve a large number of users. Other decision variables are the UAV positions and the binary auxiliary variables ϵ_{ij} (see Equation (17)).

As before, the objective is to maximize the number of users covered. Since Y_{ij} is the percentage of users in cluster j served by UAV i and U_j is the number of users in cluster j, the product $Y_{ij}\,U_j$ is the number of users in cluster j served by UAV i. Summing over UAVs yields the total number of users covered:

$$\sum_{i=1}^{n}\sum_{j=1}^{k} Y_{ij}\,U_j. \tag{12}$$

This objective function balances both the clustering approach and the practical limitations of UAV deployment while maintaining service quality across the coverage area. It also enables flexible resource allocation to serve the clusters based on their capacity.

5.2. Constraints

Since the variable Y_{ij} is a percentage, it must be constrained to the unit interval:

$$0 \le Y_{ij} \le 1, \quad \forall i \in [\![1,n]\!],\ \forall j \in [\![1,k]\!]. \tag{13}$$

Similar to the previous optimization constraints, each UAV has the capacity to serve no more than 20 users, so

$$\sum_{j=1}^{k} Y_{ij}\,U_j \le 20, \quad \forall i \in [\![1,n]\!]. \tag{14}$$

Each cluster may be served by multiple UAVs (with each UAV serving a fraction of the users), but the total percentage of users served in any cluster cannot be more than 100%, so

$$\sum_{i=1}^{n} Y_{ij} \le 1, \quad \forall j \in [\![1,k]\!]. \tag{15}$$

Previously, we allowed a UAV to serve a user only if the user was within the coverage radius of the UAV. Here, we allow a UAV to serve a fraction of users in a cluster only if the cluster center is within the coverage radius of the UAV. To formulate this constraint, we use the binary auxiliary variable ϵ_{ij} and set

$$\sqrt{(x_{\mathrm{UAV},i} - x_{\mathrm{cluster},j})^2 + (y_{\mathrm{UAV},i} - y_{\mathrm{cluser},j})^2} \le R_{\mathrm{cov}} + M\,\epsilon_{ij} \tag{16}$$

where $M > 0$ is a large constant and

$$Y_{ij} \le 1 - \epsilon_{ij}, \quad \forall i \in [\![1,n]\!],\ \forall j \in [\![1,k]\!]. \tag{17}$$

The logic behind this constraint is as follows. If the center of cluster j is not within the coverage radius of UAV i, then ϵ_{ij} must be equal to one (since it is binary, and a value of zero would violate the first condition). The second condition then implies that $Y_{ij} = 0$ so that no users in the cluster are served by the UAV. On the other hand, when the cluster center is within the coverage radius of the UAV, ϵ_{ij} may be zero, in which case $Y_{ij} \le 1$, so the UAV can serve users in the cluster.

6. Simulation Results

We now illustrate our results through several simulations. As an illustrative result, Figure 5 represents the optimized positions of $n = 5$ UAVs that maximize the number of users out of $m = 100$ with network coverage. Here, red circles denote the coverage areas of UAVs and blue dots indicate users (hotspots denote regions of higher user density—such as urban areas—and are *not* used by the algorithm). We solved the optimization problem using the CVX modeling framework [27] in Matlab with the Gurobi solver [28].

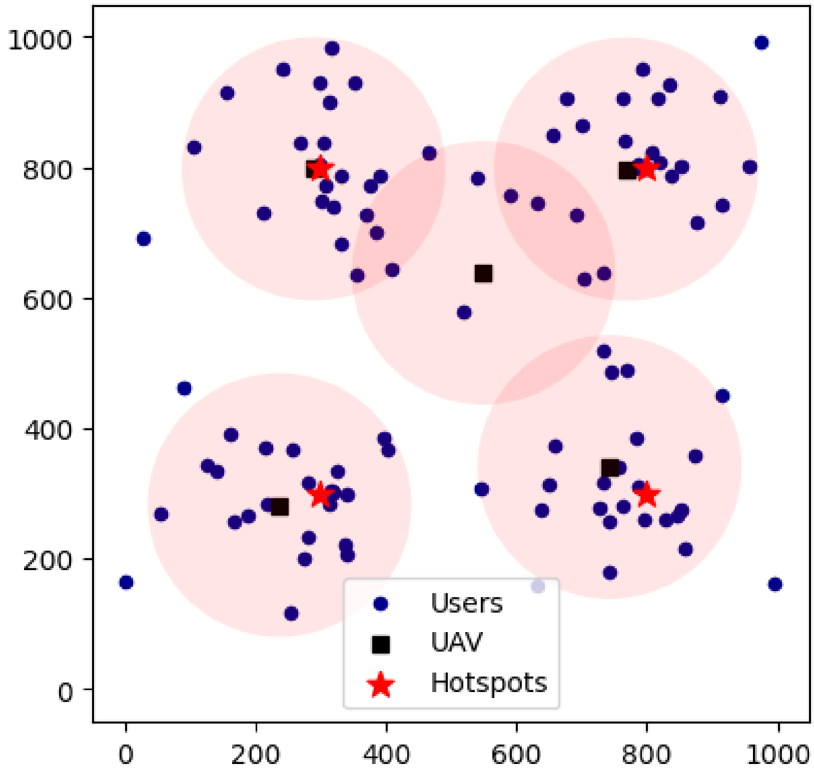

Figure 5. Positions (black squares) and coverage areas (red circles) of UAVs that maximize the number of users (blue circles) that are served. The axes indicate position in meters.

6.1. Optimal UAV Placement

We first consider the optimal placement of UAVs found by solving the MILP in Section 4. We compare the number of users served with the computational time for a varying number of UAVs with $m = 100$ users. In Figure 6, we plot the optimal value (which is the total number of users served by the group of UAVs). In Figure 7, we plot the computational time needed to solve the optimization problem for a varying number of UAVs. For $n \in \{1, 2, 3, 4\}$ UAVs, the computation time is relatively small and the optimal value increases by 20 (the capacity of each UAV) as each UAV is added. The maximum time taken was with $n = 5$ UAVs. Increasing the UAVs beyond this actually takes less time, and the total number of users increases until all $m = 100$ users are served by $n = 10$ UAVs. The results show that the coverage capacity of the UAVs increases linearly up to four UAVs, and due to the computational complexity, there is a peak and then the efficiency improves again. This non-linear behavior in time shows the trade-off between the coverage optimization and the complexity of the algorithm.

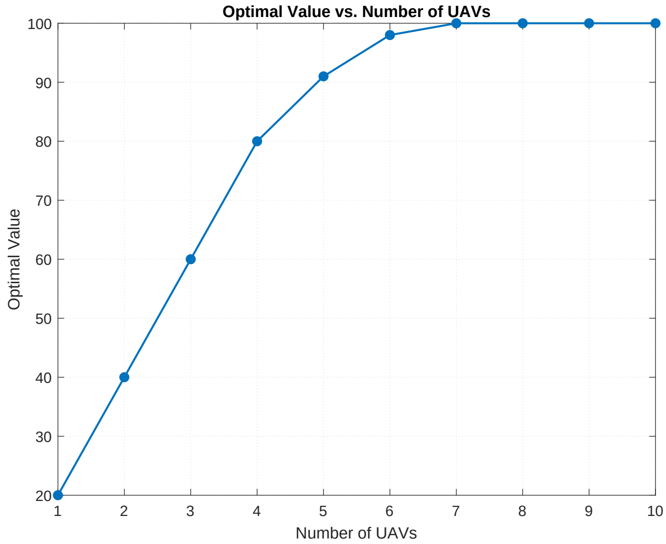

Figure 6. Number of users covered as a function of the number of UAVs. The number of users covered is the optimal value of the MILP from Section 4.

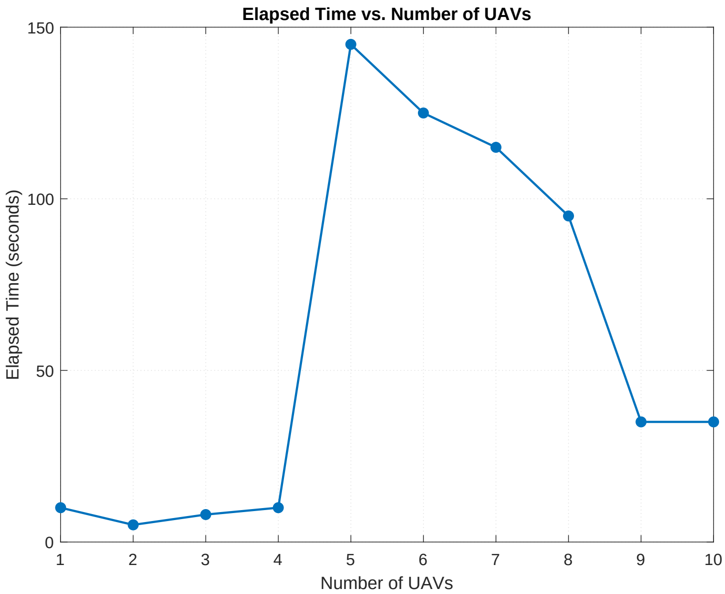

Figure 7. Time consumption to obtain the optimal solution.

6.2. Scaling to Large Numbers of Users via Clustering

To study how the problem scales with the number of users, we now consider a scenario with $m = 1000$ users. Figure 8 shows the number of users covered as a function of the number of UAVs for various numbers of clusters. Using more clusters results in a more refined characterization of the user distribution and therefore a larger number of users served. This improves more precise UAV positioning in variable user density areas. The clustering approach also shows better scalability compared to the direct optimization method for larger-scale problems. The cost of using more clusters, however, is shown in Figure 9. The computational time grows both with the number of UAVs and with the number of clusters. The computational complexity growth here follows an exponential trend that highlights the trade-off between the accuracy of the solution and the computational efficiency. We can therefore use the number of clusters to trade off the computational time needed to solve the problem with the optimality of the obtained solution.

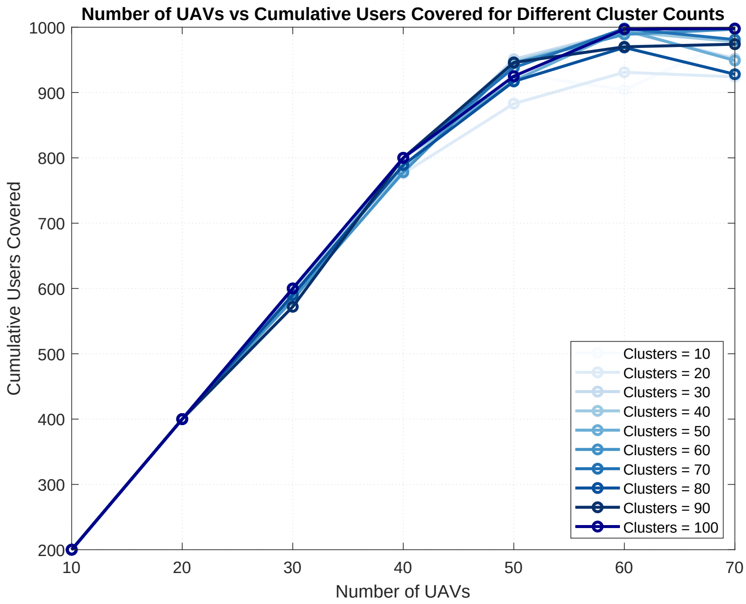

Figure 8. Total number of users covered as a function of the number of UAVs using the clustering approach.

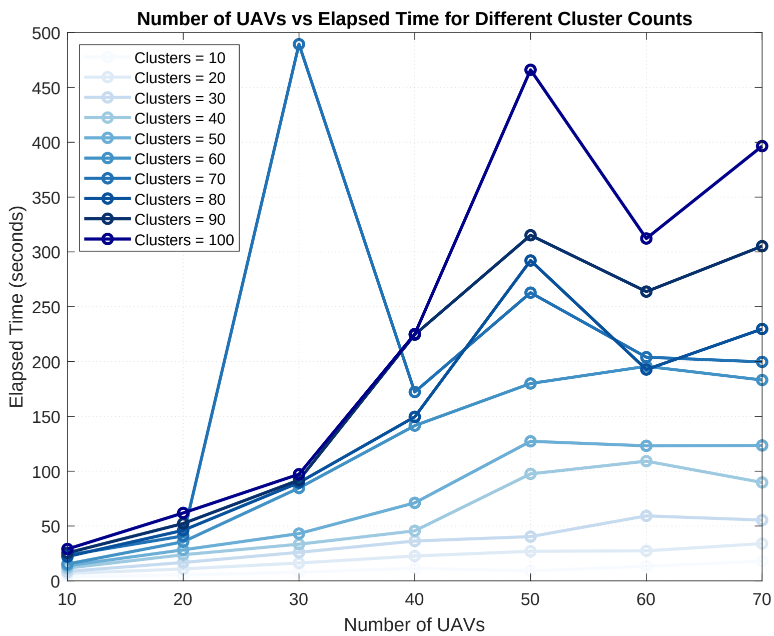

Figure 9. Computational time as a function of the number of UAVs using the clustering approach.

The combined plot including the optimal result, the results obtained using the clustering approach, and the distribution results where the UAV is randomly placed in the user grid is shown in Figure 10. Here, the general optimization approach could not solve the problem for more than 10 UAVs, as the problem grows as the number of users and UAVs increases. This limitation of the general optimization approach shows the need for efficient methods like clustering in larger-scale scenarios, which serves as a practical alternative while maintaining near-optimal performance. The distribution graph shows the random user distribution in 1000 iterations for each UAV range, and the results of the clustering approach show that the initial trend is consistent with the optimal results. Finally, we find that when the number of UAVs is 50 and the total number of users is 1000. This proves that the clustering approach is effective when the problem size increases.

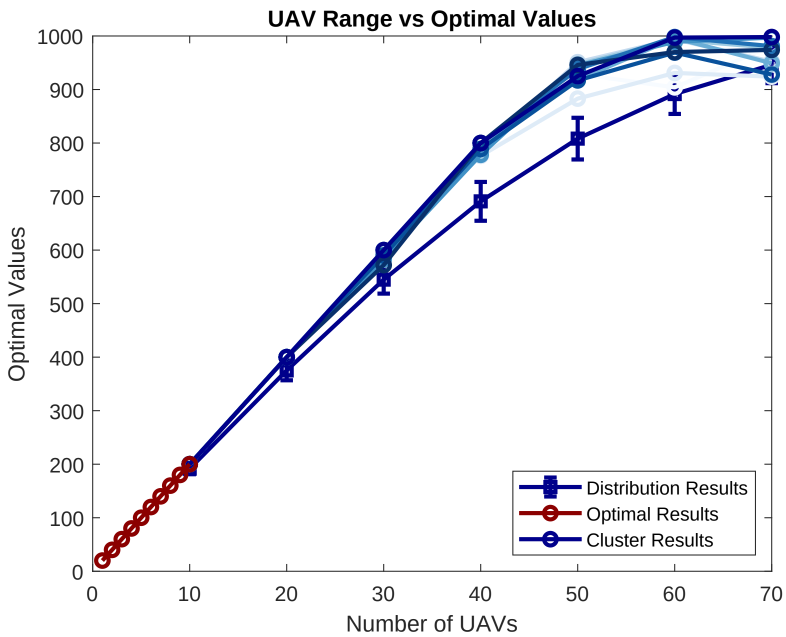

Figure 10. Combined plot for large-scale problem.

7. Conclusions

In this paper, we considered the positioning of UAV base stations in an ad hoc communication network. Specifically, we formulated the problem of positioning the UAV base stations to maximize the number of users covered as a mixed-integer linear program. For large-scale scenarios with many users, the computational complexity of finding the optimal positions can be quite high. To address this, we proposed clustering users and then solving a heuristic problem based on these clusters. The experimental results show that the number of clusters may be used to trade off user coverage and computation time. This study shows the practical application of UAVs as base stations deployed in challenging environments such as disaster scenarios and temporary network setups where network connectivity and rapid deployment are crucial. The scalability and flexibility of the clustering approach are suitable for diverse scenarios to ensure efficient resource allocation and enhanced network performance. Future work includes developing iterative algorithms capable of tracking mobile users.

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