A Fast Robust Nonlinear Dynamic Average Consensus Estimator in Discrete Time

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Introduction

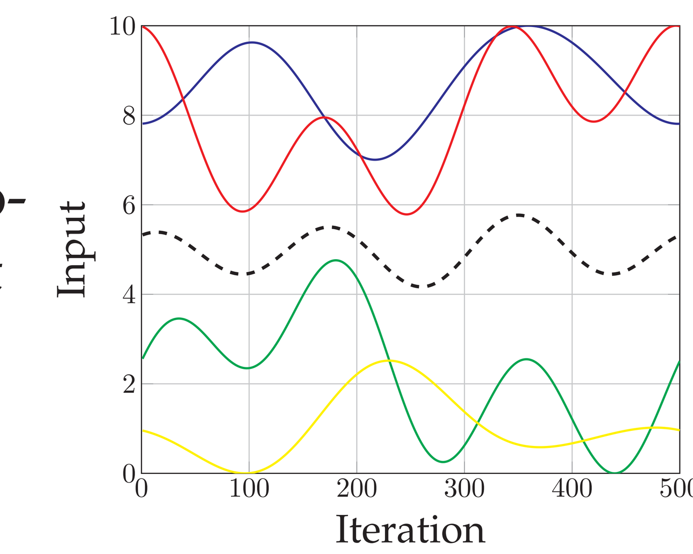
We present a new discrete-time dynamic average consensus estimator which has significant performance advantages over existing designs. It uses nonlinear oscillators to achieve both initialization robustness and internal boundedness, and its convergence rates are comparable to those of fast static consensus methods.

Estimator properties:

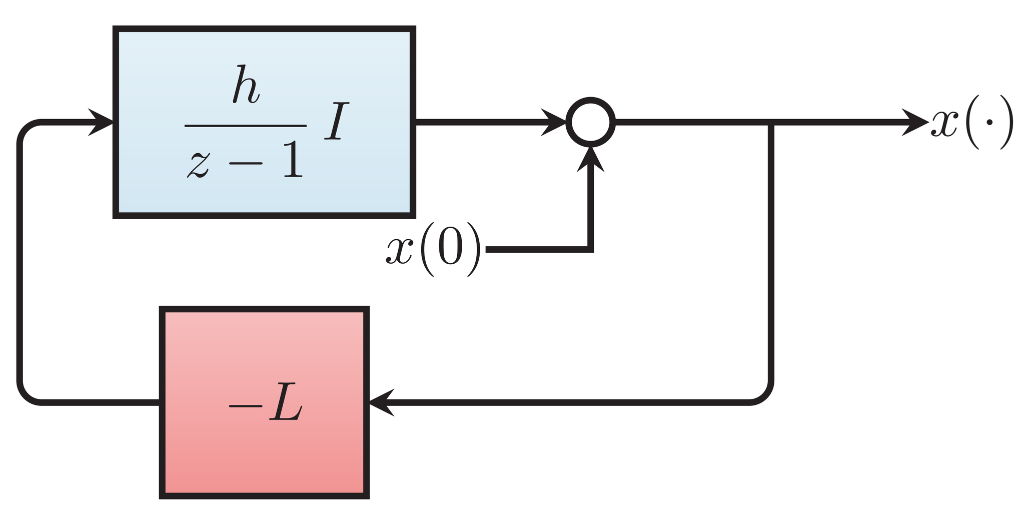
- Exactness
- Initialization robustness
- Time-invariance
- Internal stability
- Fast convergence
- Local broadcast communication

Simulations

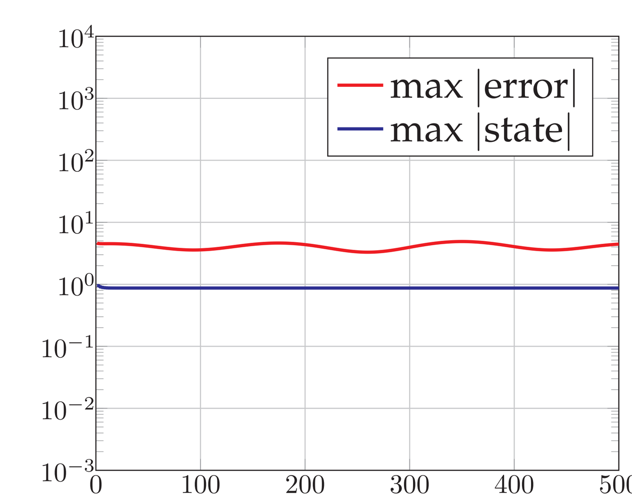
Simulations are shown for a 4-node line graph. Initial conditions are chosen randomly. The input to each agent is shown, and the black dashed line is the average to be tracked.

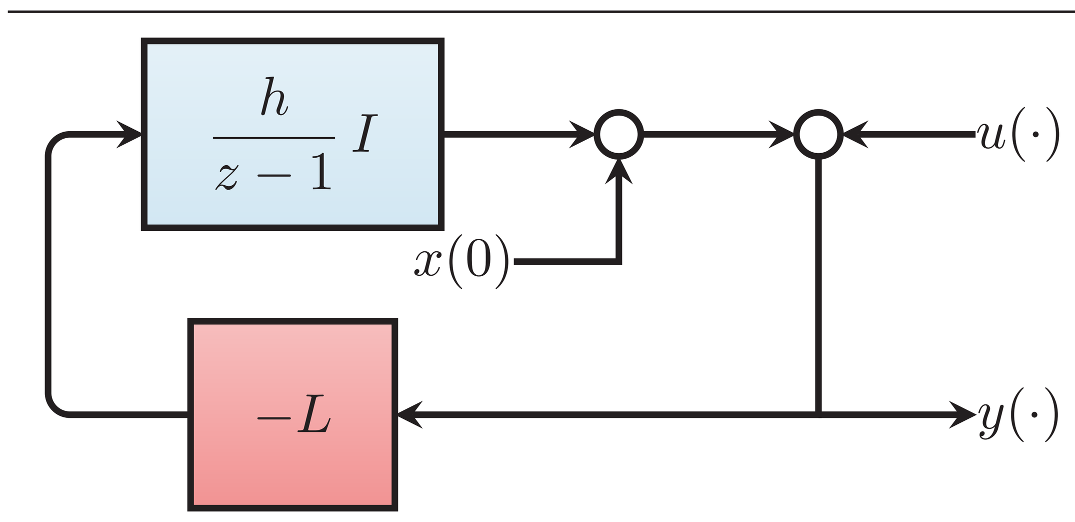


Block Diagram Progression

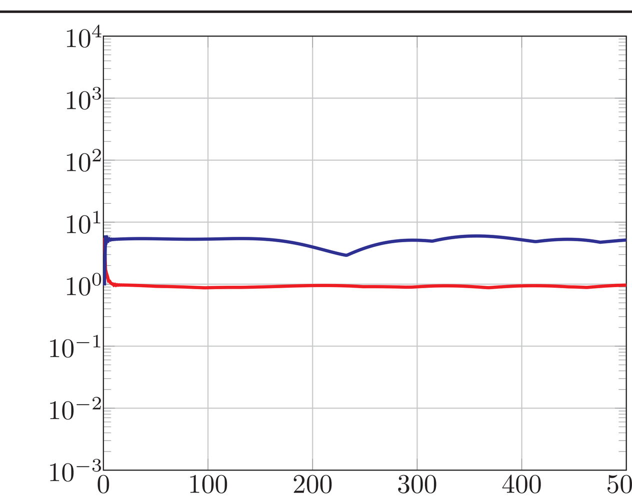


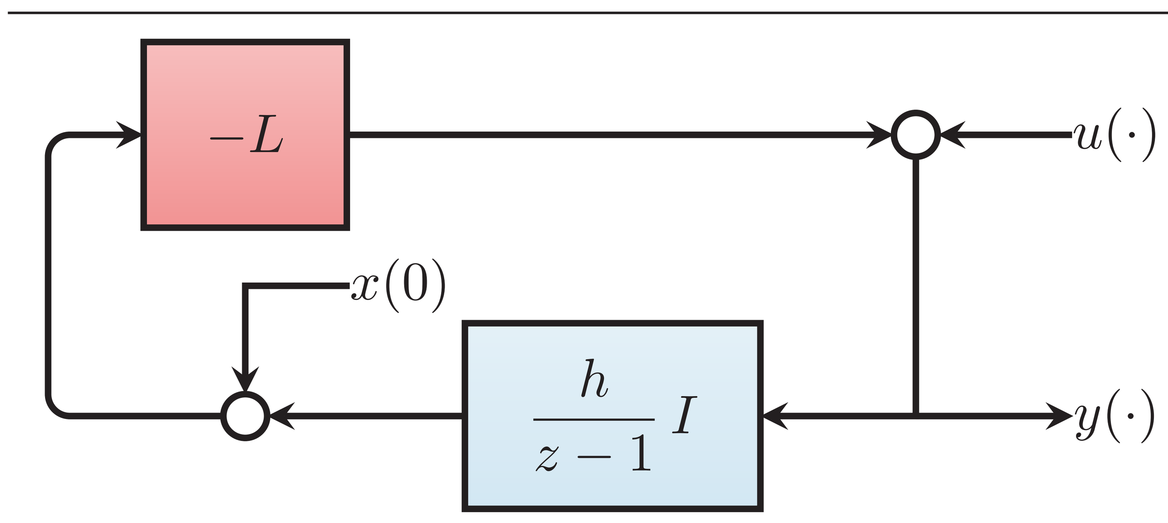
This is the static average consensus update given by
$x(k+1) = (I - hL)x(k)$
where L is the graph Laplacian and h is the step size. The estimator calculates the average of the initial state $x(0)$.
Cons: The estimator cannot track time-varying signals.
References: Tsitsiklis (1984), Xiao & Boyd (2004)



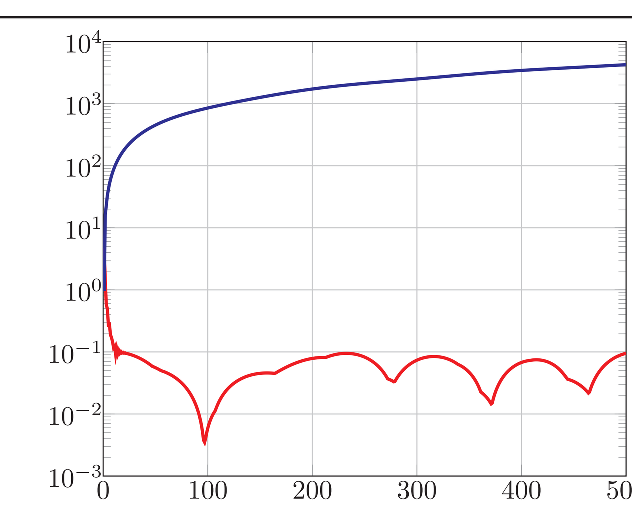


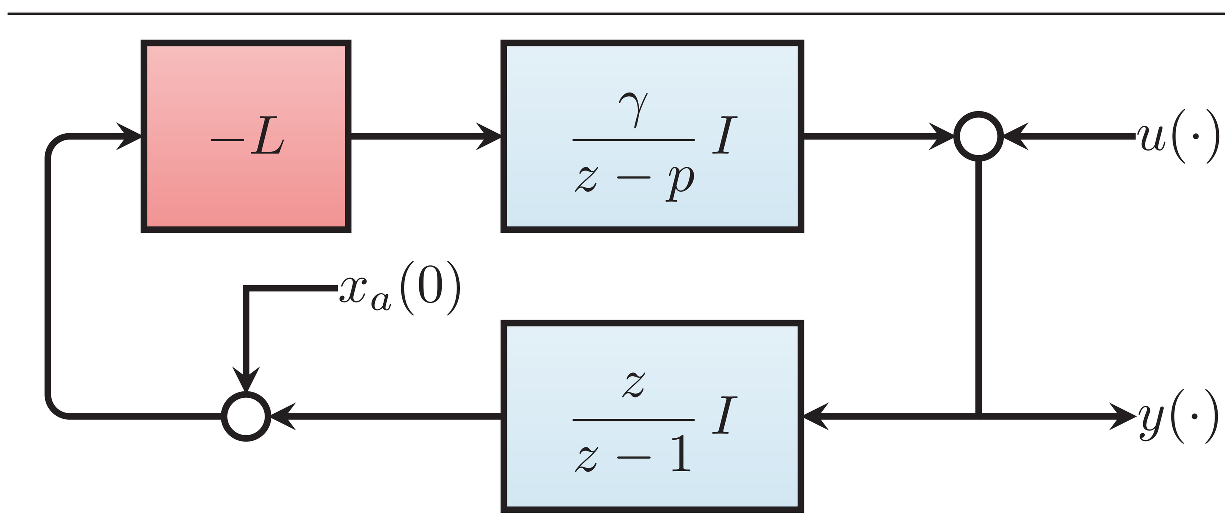
To track time-varying signals, the input signal is injected into the system as an input instead of an initial condition. For constant inputs, the output converges to $x_{\text{ave}}(0) + u_{\text{ave}}$ which depends on the initial conditions $x(0)$.
Cons: The estimator is not robust to the initial conditions.
References: Spanos et al. (2005), Zhu & Martinez (2010)



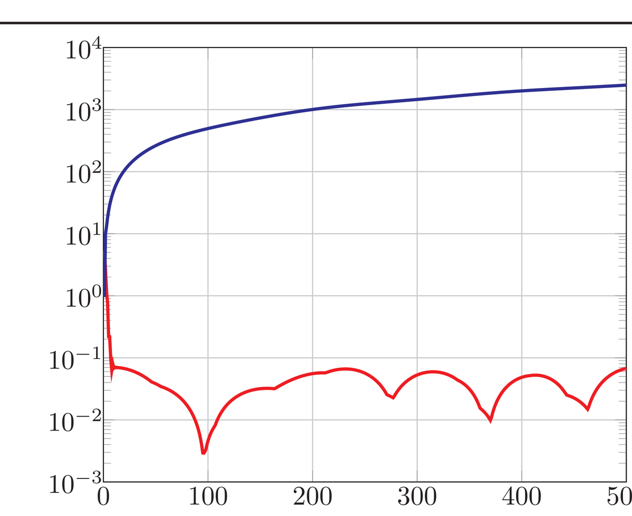


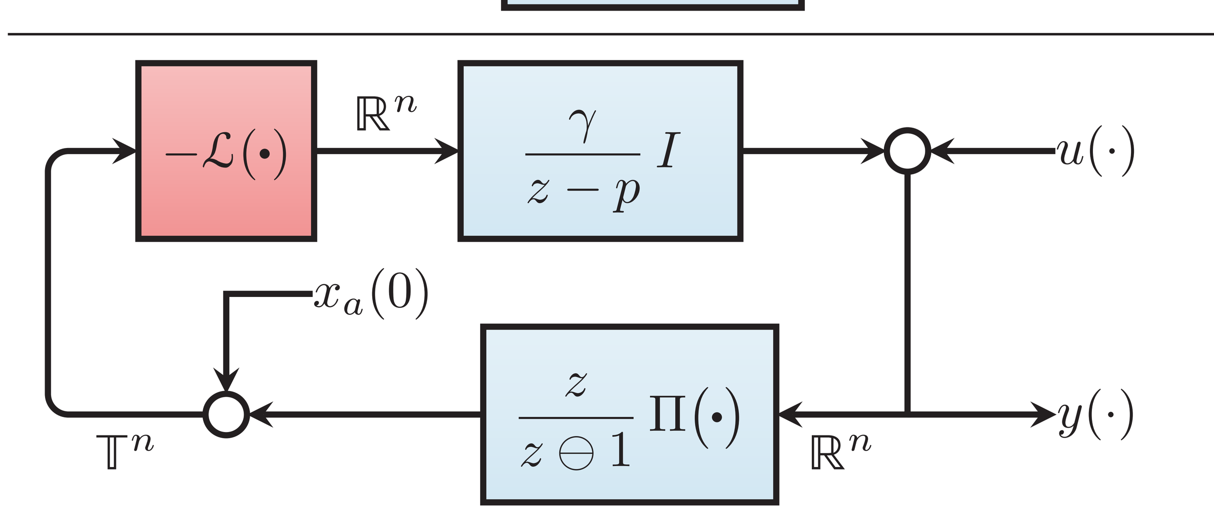
To make the estimator robust to the initial condition, the Laplacian is moved after the integrator. The initial states decay in the disagreement directions and no longer appear in the output in the consensus direction.
Cons: The current output is a function of the current states of neighboring agents, and the estimator is not internally stable.





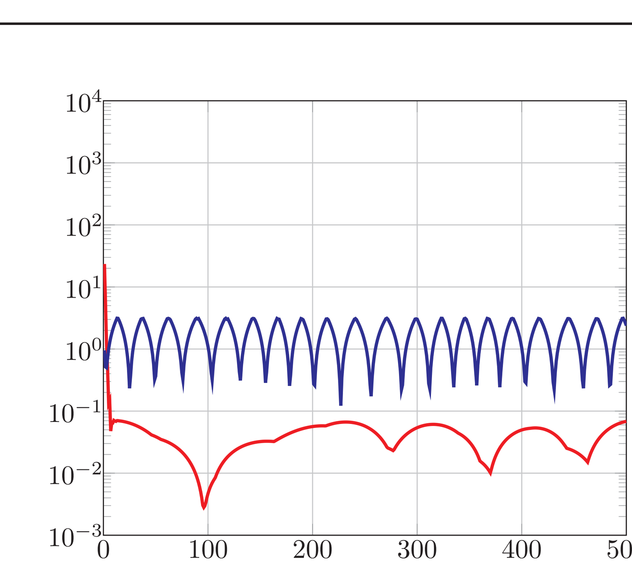
A strictly proper transfer function is inserted between the Laplacian and the output so that the current output only depends on local information. Another advantage of using an additional filter is that it can be designed to significantly reduce the time to reach convergence (see section below).
Cons: The estimator is not internally stable.



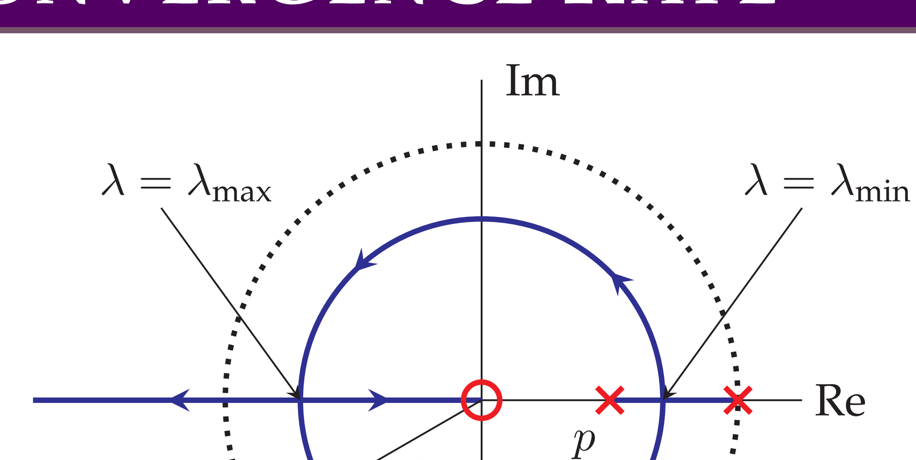


$B \in \mathbb{R}^{n\times m}$: oriented incidence matrix $\Pi : \mathbb{R}^n \to \mathbb{T}^n$
$W \in \mathbb{R}^{m\times m}$: edge weight matrix $f : \mathbb{T}^m \to \mathbb{R}^m$

To make the estimator internally stable, the state space is changed from the plane to the cylinder. The integrator states, which were previously unbounded, now live on the torus $\mathbb{T}^n$ and are inherently bounded. To do this, the nonlinear Laplacian, defined as $\mathcal{L}(\cdot) = BWf(B^T\cdot)$, is used where f is the phase coupling function and $f = I$ corresponds to the linear case, and the projection map Π replaces the identity matrix.
Cons: Cannot track signals whose average is unbounded.



Convergence Rate



Root locus in $\lambda \in \text{eig}(L)$ of $0 = 1 + \lambda\frac{\gamma}{z-p}\frac{z}{z-1}$

Filter	No	Yes
Characteristic equation	$0 = 1 + \lambda\frac{h}{z-1}$	$0 = 1 + \lambda\frac{\gamma}{z-p}\frac{z}{z-1}$
Parameters	$h = \frac{2}{\lambda_{\min}+\lambda_{\max}}$	$\gamma = \frac{(1-\alpha)^2}{\lambda_{\min}}, \quad p = \alpha^2$
Max modulus of poles	$\frac{1-\lambda_r}{1+\lambda_r}$	$\alpha = \frac{1-\sqrt{\lambda_r}}{1+\sqrt{\lambda_r}}$